Geometry: Constructions

Construction 1

Given a segment, construct a segment equal to the given segment.

Given: $\overline{AB}$

Construct: A segment equal to $\overline{AB}$

Procedure:

1. Use a straightedge to draw a line. Call it l.
2. Choose any point on l and label it X.
3. Set your compass for radius AB. Use X as center and draw an arc intersecting line l. Label the point of intersection Y.

$\overline{XY}$ is equal to $\overline{AB}$.

Justification: Since we used AB for the radius of $\odot X$, $\overline{XY}$ is equal to $\overline{AB}$.
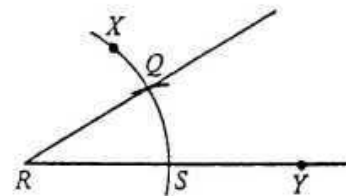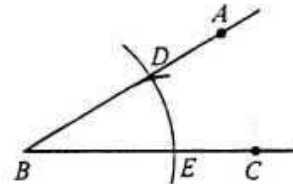
Construction 2

Given an angle, construct an angle equal to the given angle.

Given: $\angle ABC$

Construct: An angle equal to $\angle ABC$

Procedure:

1. Draw a ray. Label it $\overrightarrow{RY}$.
2. Using B as center and any convenient radius, draw an arc intersecting $\overrightarrow{BA}$ and $\overrightarrow{BC}$. Label the points of intersection D and E.
3. Using R as center and the same radius as before, draw an arc intersecting $\overrightarrow{RY}$ and label it $\overset{\frown}{XS}$, with S at the point of intersection.
4. Using S as center and a radius equal to DE, draw an arc that intersects $\overset{\frown}{XS}$ at a point Q.
5. Draw $\overrightarrow{RQ}$.

$\angle R$ is equal to $\angle B$.

Justification: If $\overline{DE}$ and $\overline{QS}$ are drawn, $\triangle DBE \cong \triangle QRS$ (SSS Postulate). Then $\angle R = \angle B$.
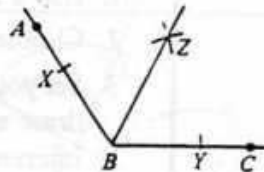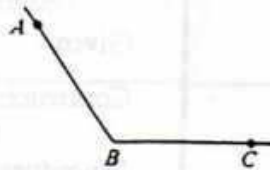
Construction 3 Given an angle, bisect the angle.

Given: $\angle ABC$

Construct: The ray that bisects $\angle ABC$

Procedure:

1. Using B as center and any convenient radius, draw arcs intersecting $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in points X and Y.
2. Using X and Y as centers and any convenient radius, draw arcs that intersect at a point Z.
3. Draw $\overrightarrow{BZ}$.

$\overrightarrow{BZ}$ bisects $\angle ABC$.

Justification: If $\overline{XZ}$ and $\overline{YZ}$ are drawn, $\triangle XBZ \cong \triangle YBZ$ (SSS Postulate). Then $\angle XBZ = \angle YBZ$, and $\overrightarrow{BZ}$ bisects $\angle ABC$.

Construction 4 Given a point on a line, construct the perpendicular to the line at the given point.
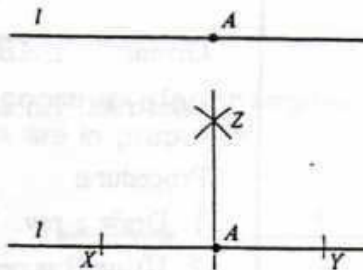
Given: Point A on line l

Construct: The perpendicular to l at A

Procedure:

Bisect the straight angle whose vertex is A.

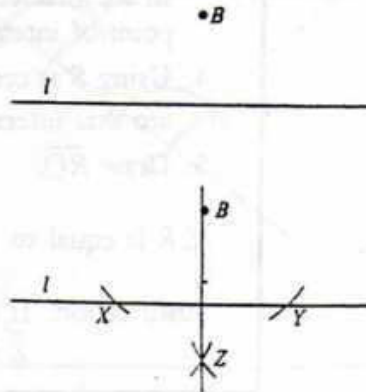$\overleftrightarrow{AZ}$ is perpendicular to l at A.

Construction 5 Given a point outside a line, construct the perpendicular to the line from the point.

Given: Point B outside line l

Construct: The perpendicular to l from B

Procedure:

1. Using B as center and any convenient radius, draw arcs that intersect l in two points X and Y.
2. Using X and Y as centers and any convenient radius, draw arcs that intersect at a point Z.
3. Draw $\overrightarrow{BZ}$.

$\overleftrightarrow{BZ}$ is perpendicular to l.

Construction 6 Given a segment, construct the perpendicular bisector of the segment.

Given: $\overline{CD}$

Construct: The perpendicular bisector of $\overline{CD}$

C •————————————————————• D

Procedure:

1. Using any convenient radius, construct two arcs having C as center and two arcs having D as center. Call the points of intersection X and Z.

2. Draw $\overline{XZ}$.

$\overline{XZ}$ is the perpendicular bisector of $\overline{CD}$.